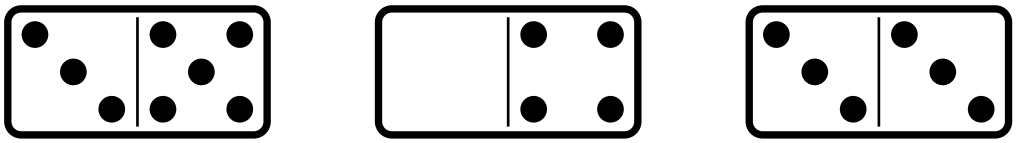
Problem of the Week

Problem D

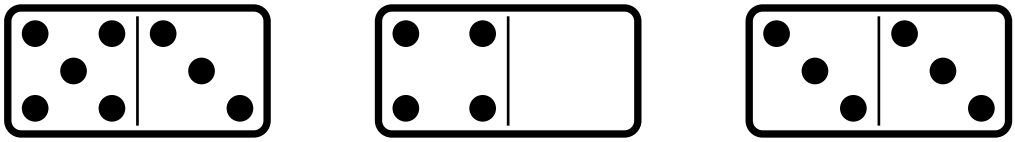
Something's Missing

A domino tile is a rectangular tile with a line dividing its face into two square ends. Each end is marked with a number of dots (also called pips) or is blank.



The domino on the left is a $[3, 5]$ domino, since there are 3 pips on one end and 5 pips on the other end. The domino in the middle is a $[0, 4]$ domino, since there are 0 pips on one end and 4 pips on the other end. The domino on the right is a $[3, 3]$ domino, since there are 3 pips on one end and 3 pips on the other end. Note, this domino is also called a double domino since both sides have the same number of pips.

We can also rotate the domino tiles:



The domino on the left is a $[5, 3]$ domino. However, since each tile has just been rotated, $[5, 3]$ and $[3, 5]$ represent the same domino. Similarly, the domino in the middle is a $[4, 0]$ domino. Note that $[4, 0]$ and $[0, 4]$ represent the same domino.

An n-set of dominoes contains all the tiles with the number of pips on any end ranging from 0 to n, and no two dominoes can be the same.

For example, a 2-set of dominoes has the following 6 tiles: $[0, 0], [0, 1], [0, 2], [1, 1], [1, 2], [2, 2]$. (Notice that the three dominoes $[1, 0]$, $[2, 0]$ and $[2, 1]$ are not listed because they are the same as the three dominoes $[0, 1]$, $[0, 2]$ and $[1, 2]$).

Dominic and Dmitri divide a 6-set of dominoes into two unequal piles. They then realize that one domino is missing from the set. Dimitri counts all of the pips on the dominoes in the first pile. He counts that there are a total of 91 pips. Dominic counts all of the pips on the dominoes in the second pile. He counts that there are a total of 67 pips. Dominic also notes that all the double dominoes are accounted for. Which domino is missing from the set?